Heat transfer modes

conduction

Fourier's Law

$q'' = -k \frac{dT}{dx}$

(W) $q = -k.A \frac{dT}{dx}$

k: thermal conductivity
material property
$\left(\frac{W}{mK}\right)$

- Conduction occurs in stagnant medium (no motion)

⇓

Energy balances + Boundary conditions

⇓

we can reach to solution.

For ex: Heat transfer from extended surfaces ⇒ mathematical solutions are tabulated on Table 3.4 (T distribution)
Fin heat tr rate

extended surface enables higher heat transfer rates due to increased area.

η = fin efficiency ⇒ Figures (3.18 – 3.19) & Table 3.5

$= \frac{\text{heat dissipated from the fin surface}}{\text{heat which will be dissipated if the fin } T = T_b \text{ everywhere}}$

convection

Newton's Law of cooling

$q'' = h.(T_\infty - T_s)$

$q = h.A(T_\infty - T_s)$

h: convection coefficient
depends on
- fluid type
- flow regime
- velocity
- geometry

$h\left[\frac{W}{m^2K}\right]$

- convection occurs on a surface and a fluid on motion. + large number of molecules are moving as aggregates, collectively.

⇓

$\boxed{\text{EXPERIMENT F}}$

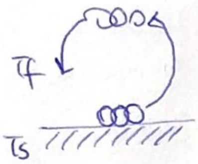
radiation

Stefan Boltzman Law

$q = \sigma A (T^4 - T_s^4)$
↓
ε for gray bodies

Convection (EXPERIMENT F)

natural or free convection



$T_s > T_f$

→ originates in case of temperature gradients

→ T gradients cause gradients in density (ρ)

- buoyancy force is the net effect
- body forces → usually gravitational

↑ buoyancy forces

↓ body forces

Free convection ⇒ flow velocities are much smaller

⇒ convection heat tr. rates are also smaller

⇒ larger resistance to heat tr (can be useful if we want to minimize heat tr)

density depends on T

$\rho \uparrow$ when $T \downarrow$

$\rho \downarrow$ when $T \uparrow$ due to fluid expansion

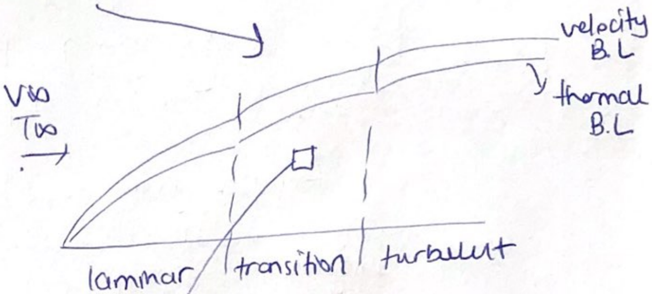
$$PV = NRT$$

$$m = \rho V$$

forced convection

we need external equipment

- blowers
- fan
- pump
- (winds)



we need to solve

1. conservation of mass
2. " " energy
3. " " momentum

on a differential element

⇓

difficult to solve

⇓

instead we can use empirical solutions to solve the problems and find $\underline{\underline{h}}$

⇓

introduce similarity parameters and dimensionless numbers

$x^* = \frac{x}{D}$ or $\frac{x}{L}$

L: characteristic length

$T^* = \frac{T - T_0}{T_1 - T_0}$

FORCED CONV.

$$Nu_L = h \cdot \frac{L}{k} \qquad Nu_D = h \frac{D}{k} \text{ (for a cylinder)}$$

For a vertical plate

$h = Nu_L \frac{k}{L}$

k → thermal conductivity of the fluid $\left(\frac{W}{mK}\right)$

L → length of the plate (m)

Nu_L → Nusselt number

$$Re = \frac{L \cdot V \rho}{\mu} = \frac{L V}{\nu} \cong \frac{\text{inertial f.}}{\text{viscous f}}$$

μ → viscosity

ν → kinematic viscosity $= \mu / \rho$

$$Re = \frac{[m]\left[\frac{m}{s}\right]\left[\frac{kg}{m^3}\right]}{\left[\frac{kg}{ms}\right]}$$

$$\boxed{Re_c = 5 \times 10^{-5}}$$

Laminar Flow

$$Nu_x = \frac{0.3387\, Re_x^{1/2}\, Pr^{1/3}}{\left[1 + (0.0468/Pr)^{2/3}\right]^{1/4}} \quad \text{if } Pe_x \gtrsim 100$$

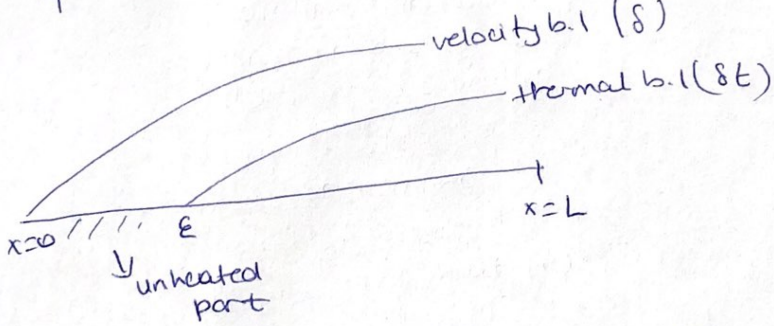
$$\text{Prandl number} = Pr = \frac{c_p \cdot \mu}{k} = \frac{\nu}{\alpha} = \frac{\text{momentum}}{\text{thermal diff.}}$$

$$Pe = Re \cdot Pr = \frac{V \cdot L}{\alpha} \cong \frac{\text{conv}}{\text{cond}}$$

Laminar + Turbulent Flow

$$Nu_x = 0.0296\, Re^{4/5}\, Pr^{1/3} \quad \text{if } 0.6 \lesssim Pr \lesssim 60$$

if there is an unheated part on the surface ⇒ boundary layers will initiate at different locations



⇓

we can use a corrected Nu number

$$\overline{Nu_L} = Nu_L\Big|_{\varepsilon=0} \cdot \frac{L}{L-\varepsilon}\left[1 - \left(\frac{\varepsilon}{L}\right)^{(p+1)/(p+2)}\right]^{p/p+1} \qquad \text{Eqn 7.44}$$

$p = 2$ for laminar flow

$p = 8$ " turbulent "

In order to use the ~~[illegible]~~ dimensionless numbers ⇒ we need fluid properties

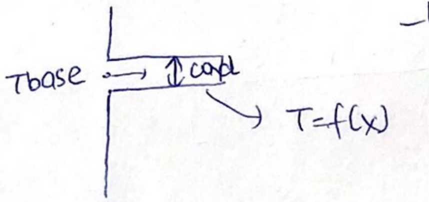
Fluid properties in general based on film T ⇒ $T_f = \frac{T_s + T_\infty}{2}$

$q = h \cdot A \, (T_s - T_f)$ ⇒ To increase heat tr from the surface to surrounding atm.

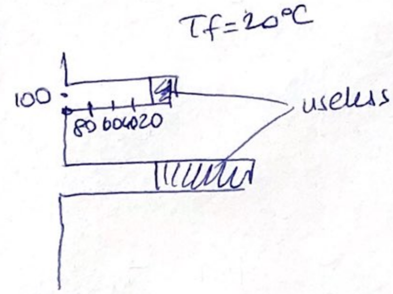
$h \uparrow$ expensive, and sometimes obtained h_{max} is not enough with the pumps & blowers

$T_f \downarrow$ not practical

$A \uparrow$ fins & pins & extended surfaces



$$-k\frac{dT}{dx} = h(T - T_\infty)$$



in case of pins ⇒ there will be a resistance to convection
we will have extra resistance

forced convection or free convection. $\overline{Nu} = f(Re, Pr, Gr)$

$$Gr = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}$$

β = thermal expansion coefficient
$= 1/T$ for an ideal gas

free conv $\quad \frac{Gr}{Re^2} \gg 1 \quad \overline{Nu} = f(Gr, Pr)$

forced " $\quad \frac{Gr}{Re^2} \ll 1 \quad \overline{Nu} = f(Re, Pr)$

free & forced $\quad \frac{Gr}{Re^2} \cong 1 \quad \overline{Nu} = f(Re, Pr, Gr)$

Free convection ⇒ $Ra = Gr \cdot Pr = \frac{g \cdot \beta(T_s - T_\infty)L^3}{\nu\alpha}$

$Nu = c \cdot Ra^a$ Eqns (9.30 – 9.32)